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# Multicriteria Decision-making Method Based on Risk Attitude under Interval-valued Intuitionistic Fuzzy Environment

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**Abstract** Based on the feature of interval-valued intuitionistic fuzzy multi-attribute decision-making, in this thesis, a mentality parameter is used to reflect the decision makers' risk attitude in determining of both a membership degree and a non-membership degree. Besides, with the mentality parameter, a new score function and accuracy function are proposed, which integrate the membership degree, the non-membership degree and the hesitancy degree into one index. Furthermore, to compare two interval-valued intuitionistic fuzzy numbers, a new ranking method is generated with the score function and accuracy function. Finally, a multi-attribute decision method under interval-valued intuitionistic fuzzy environment is developed in a linear weighted average operator. And promising numerical results show that this method is available.

**Keywords** Interval-valued intuitionistic fuzzy numbers · Score function · Mentality parameter · Multi-attribute decision making


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## 1. Introduction

In 1986, intuitionistic fuzzy sets (IFSs) were proposed by Atanassov [1], which are an extension of fuzzy sets introduced by Zadeh [2]. Then, Atanassov et al. presented intuitionistic fuzzy interpretation of multi-person multi-criteria decision making [3, 4], and intuitionistic fuzzy interpretations of the processes of multi-person and of multi-measurement tool and multi-criteria decision makings are discussed [5]. Furthermore, Pasi et al. proposed intuitionistic fuzzy interpretations of elements of utility [6]. In IFSs, the membership degree, the non-membership degree and the hesitancy degree are considered. Comparing with traditional fuzzy sets, they show more flexibility and practicality in dealing with the uncertainty of the objectives. Such a generalization of fuzzy sets gives us an additional possibility to represent imperfect knowledge that leads to describing many real problems in a more adequate way. Szmidt et al. provided a solution to a multi-criteria decision making problem by using similarity measures for IFSs [7]. Later, Szmidt et al. proposed a new method of IFSs which takes into account not only the amount of information related to an alternative (expressed by a distance from an ideal positive alternative) but also the reliability of information represented by an alternative meant as how sure the information is [8, 9]. And Szmidt et al. presented some of the extended decision making models and showed why IFSs make it possible to avoid some more common cognitive biases [10]. Recently, applications of IFSs to multi-criteria fuzzy decision making are presented. For example, Liu and Wang provided a new method to hand multi-criteria fuzzy decision making problems based on IFSs [11]. Chen presented a new approach for solving problem by using decision tree induction based on IFSs [12]. Ye proposed a fuzzy multi-criteria decision making method based on weighted correlation coefficients by using entropy weights under intuitionistic fuzzy environment [13]. Zhang and Xu proposed a new method for ranking intuitionistic fuzzy values (IFVs) by using the similarity measure and the accuracy degree [14]. However, for IFSs, it is difficult to determine the membership degree and the non-membership degree as exact numbers. That is, it is impossible to estimate their ranges. Atanassov and Gargov thus introduced the concept of interval-valued intuitionistic fuzzy sets (IVIFSs) [15], whose membership and non-membership are a closed interval. IVIFSs are more exquisite and accurate in depicting uncertainty of things. With the development of IFSs theory, IVIFSs as an extension of IFSs were also applied to decision-making problems. In multi-attribute fuzzy decision-making problems, it emphasizes that ranking of IVIFSs is a key to determining an optimal one from multiple alternatives, therefore, the study on ranking methods of IVIFSs has become the most important step during decision-making process. Chen and Tan presented a score function for handing multi-attribute fuzzy decision-making problems based on vague set theory [16]. And then Hong and Choi indicated that the score function cannot discriminate some alternatives, and provided the accuracy function [17]. Xu presented a method for ranking IFVs based on the score function and the accuracy function and it was extended to IVIFSs [18]. Xu studied the method of interval-valued intuitionistic fuzzy multi-criteria decision-making and proposed a score function and an accuracy function for IVIFSs [19]. However, focusing on the information of membership and non-membership, Xu paid little attention to the information of hesitancy. Interval-valued intuitionistic fuzzy numbers (IVIFNs) mainly consist of the membership interval, non-membership interval and hesitancy interval, therefore, the ranking index of IVIFNs must make full use of the information of these three aspects. Ye [20], Lee [21] and Lakshmana [22] proposed novel functions for IVIFSs by taking the hesitancy information of IVIFSs into account to overcome the deficiency of Xu's ranking method. Wang [23] added two ranking functions by considering the uncertainty of membership and non-membership. However, in some cases, we can not identify an optimal alternative although the alternatives are apparently different by applying the novel function proposed by Ye, Lee and Lakshmana. And sometimes the ranking result obtained by Wang's method is not reasonable. In addition, the decision results are highly affected by a decision maker with risk attitude. For example, a risk-averse decision maker would like to select low risk alternatives while a risk-seeking one would like high risk ones. Therefore, in this paper, a new score function is proposed to properly reflect the decision maker, based on discussion of hesitation in IVIFNs. Compared with existing methods, a method proposed in this paper is able to flexibly reflect his risk attitude and the information of IVIFNs can be fully utilized.

The rest of this thesis is organized as follows. Section 2 presents some basic concepts of IVIFSs. In Section 3, a new score function and accuracy function are proposed. A new multi-attribute decision making with IVIFNs is generated in Section 4. In Section 5, a numerical example is discussed. Finally, the conclusions are presented in Section 6.

## 2. Preliminaries

As a preparation for introducing our new method, some basic concepts of IVIFSs are illustrated in this section.

**Definition 2.1** [15] *Let $X$ be an ordinary finite non-empty set. An interval-valued intuitionistic fuzzy set (IVIFS) $\tilde{A}$ in $X$ is an expression with the form*

$$\tilde{A} = \{\langle x, \mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x)\rangle | x \in X\}, \tag{1}$$

*where the functions $\mu_{\tilde{A}}(x) : X \to D[0,1]$ and $\nu_{\tilde{A}}(x) : X \to D[0,1]$ denote the degree of membership and the degree of non-membership of the element $x \in X$ in $\tilde{A}$, respectively. $\mu_{\tilde{A}}(x)$ and $\nu_{\tilde{A}}(x)$ are closed intervals and their lower and upper boundaries are denoted by $\mu_{\tilde{A}L}(x)$, $\mu_{\tilde{A}U}(x)$, $\nu_{\tilde{A}L}(x)$ and $\nu_{\tilde{A}U}(x)$, respectively. We can denote $\tilde{A}$ by*

$$\tilde{A} = \{\langle x, [\mu_{\tilde{A}L}(x), \mu_{\tilde{A}U}(x)], [\nu_{\tilde{A}L}(x), \nu_{\tilde{A}U}(x)]\rangle | x \in X\} \tag{2}$$

*with $0 \le \mu_{\tilde{A}U}(x) + \nu_{\tilde{A}U}(x) \le 1$, $\mu_{\tilde{A}L}(x) \ge 0$, $\nu_{\tilde{A}L}(x) \ge 0$ .*

**Definition 2.2** [15] *For each IVIFS $\tilde{A}$ in $X$, if*

$$\pi_{\tilde{A}}(x) = 1 - \mu_{\tilde{A}}(x) - \nu_{\tilde{A}}(x) = [1 - \mu_{\tilde{A}U}(x) - \nu_{\tilde{A}U}(x), 1 - \mu_{\tilde{A}L}(x) - \nu_{\tilde{A}L}(x)], \tag{3}$$

*then* $\pi_{\tilde{A}}(x)$ *is called a hesitancy degree of an intuitionistic fuzzy interval of* $x \in X$ *in* $\tilde{A}$ *.*

Especially, if $\mu_{\tilde{A}L}(x) = \mu_{\tilde{A}U}(x)$ and $\nu_{\tilde{A}L}(x) = \nu_{\tilde{A}U}(x)$, then IVIFS $\tilde{A}$ reduces to an intuitionistic fuzzy set (IFS).

For IVIFS $\tilde{A}$ and a given $x$, the pair $(\mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x))$ is called an IVIFN [24]. For convenience, the pair $(\mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x))$ is often denoted by $\tilde{\alpha} = ([a, b], [c, d])$, where $[a, b] \subset D[0, 1]$, $[c, d] \subset D[0, 1]$ and $b + d \leq 1$. For IVIFN $\tilde{\alpha} = ([0.6, 0.7], [0.1, 0.2])$, its physical interpretation can be expressed as "for an election with 100 voters, and it is expected that there are 60 – 70 in favor, 10 – 20 against, and 10 – 30 abstentions".

Let $\tilde{\alpha} = ([a, b], [c, d])$ be an IVIFN, middle value of membership interval, non-membership interval and hesitancy interval are defined by the following formula respectively:

$$M(\mu_{\tilde{\alpha}}) = \frac{a+b}{2}, M(\nu_{\tilde{\alpha}}) = \frac{c+d}{2}, M(\pi_{\tilde{\alpha}}) = \frac{(1-b-d)+(1-a-c)}{2},$$

where $M(\mu_{\tilde{\alpha}}) + M(\nu_{\tilde{\alpha}}) + M(\pi_{\tilde{\alpha}}) = 1$.

Xu introduced some basic arithmetical operations and relations [19], which are useful in the remainder of this paper.

**Definition 2.3** [19] *Let* $\tilde{\alpha} = ([a_1, b_1], [c_1, d_1])$ *and* $\tilde{\beta} = ([a_2, b_2], [c_2, d_2])$ *be two IVIFNs. Then their operational laws are defined as follows:*

$$\tilde{\alpha} \geq \tilde{\beta} \; if \; a_1 \geq a_2, b_1 \geq b_2, c_1 \leq c_2 \; and \; d_1 \leq d_2; \tag{4}$$

$$\tilde{\alpha} = \tilde{\beta} \; if \; a_1 = a_2, b_1 = b_2, c_1 = c_2 \; and \; d_1 = d_2. \tag{5}$$

## 3. Score Function and Accuracy Function

A score function and an accuracy function defined by Xu were used to rank IVIFNs [19]. However, the ranking method did not give sufficient information about alternatives. Therefore, we will present a new method to rank IVIFNs in this section.

**Definition 3.1** [19] *Let* $\tilde{\alpha} = ([a, b], [c, d])$ *be an IVIFN. Its score function and accuracy function can be denoted as:*

$$S(\tilde{\alpha}) = \frac{a-c+b-d}{2}, \tag{6}$$

$$h(\tilde{\alpha}) = \frac{a+b+c+d}{2}. \tag{7}$$

Obviously, $S(\tilde{\alpha}) = M(\mu_{\tilde{\alpha}}) - M(\nu_{\tilde{\alpha}}) \in [-1, 1], h(\tilde{\alpha}) = M(\mu_{\tilde{\alpha}}) + M(\nu_{\tilde{\alpha}}) \in [0, 1]$ and the bigger the score of $\tilde{\alpha}$, the larger the interval-valued intuitionistic fuzzy value (IVIFV) $\tilde{\alpha}$.

Based on the score function and accuracy function, a prioritized comparison method is introduced as follows.

**Definition 3.2** [19] *Let $\tilde{\alpha}$ and $\tilde{\beta}$ be two IVIFNs, $S(\tilde{\alpha})$, $S(\tilde{\beta})$ be the scores of $\tilde{\alpha}$ and $\tilde{\beta}$, and $h(\tilde{\alpha})$, $h(\tilde{\beta})$ be the accuracy degrees of $\tilde{\alpha}$ and $\tilde{\beta}$, respectively. Then:*

1) *If $S(\tilde{\alpha}) < S(\tilde{\beta})$, then $\tilde{\alpha}$ is smaller than $\tilde{\beta}$, denoted by $\tilde{\alpha} \prec \tilde{\beta}$.*

2) *If $S(\tilde{\alpha}) > S(\tilde{\beta})$, then $\tilde{\alpha}$ is larger than $\tilde{\beta}$, shown as $\tilde{\alpha} \succ \tilde{\beta}$.*

3) *If $S(\tilde{\alpha}) = S(\tilde{\beta})$, then :*

i) *if $h(\tilde{\alpha}) < h(\tilde{\beta})$, $\tilde{\alpha}$ is smaller than $\tilde{\beta}$, denoted by $\tilde{\alpha} \prec \tilde{\beta}$;*

ii) *if $h(\tilde{\alpha}) > h(\tilde{\beta})$, $\tilde{\alpha}$ is larger than $\tilde{\beta}$, denoted by $\tilde{\alpha} \succ \tilde{\beta}$;*

iii) *if $h(\tilde{\alpha}) = h(\tilde{\beta})$, $\tilde{\alpha}$ is equivalent $\tilde{\beta}$, shown as $\tilde{\alpha} \sim \tilde{\beta}$.*

However, Xu focused on the information of membership and non-membership paying little attention to the information of hesitancy. So, in some cases, we can not identify an optimal alternative by applying Definition 3.1. By noticing the limitations of Xu's ranking method, Ye [20], Lee [21] and Lakshmana [22] introduced new score function by considering the hesitancy degree and Wang [23] added two ranking functions by considering the uncertainty of membership and non-membership.

**Definition 3.3** [20] *Let $\tilde{\alpha} = ([a, b], [c, d])$ be an IVIFN. Its accuracy function can be denoted as:*

$$S_Y(\tilde{\alpha}) = a + b - 1 + \frac{c + d}{2}. \tag{8}$$

**Definition 3.4** [21] *Let $\tilde{\alpha} = ([a, b], [c, d])$ be an IVIFN. Its score function can be denoted as:*

$$S_L(\tilde{\alpha}) = \frac{2 + a + b - c - d}{3 - a - b - c - d}. \tag{9}$$

**Definition 3.5** [22] *Let $\tilde{\alpha} = ([a, b], [c, d])$ be an IVIFN. Its accuracy function can be denoted as:*

$$S_A(\tilde{\alpha}) = \frac{1}{2}[a + b - d(1 - b) - c(1 - a)]. \tag{10}$$

**Definition 3.6** [23] *Let $\tilde{\alpha} = ([a, b], [c, d])$ be an IVIFN. Its membership uncertainty index and non-membership uncertainty index can be denoted as:*

$$T(\tilde{\alpha}) = b + c - a - d, \tag{11}$$

$$G(\tilde{\alpha}) = b + d - a - c. \tag{12}$$

In some cases, however, the ranking result obtained by these ranking methods [19-23] is not reasonable.

*Example 1* Given interval-valued intuitionistic fuzzy values (IVIFVs) for two alternatives:

$$\tilde{\alpha}_1 = ([0.3, 0.5], [0.2, 0.4]), \tilde{\alpha}_2 = ([0.35, 0.45], [0.25, 0.35]).$$

By applying Eqs. (6)-(9), we can obtain $\tilde{\alpha}_1 \sim \tilde{\alpha}_2$, so we cannot get the order of these two alternatives. By using Eq. (10), we obtain that $\tilde{\alpha}_1 \succ \tilde{\alpha}_2$. By using Eqs. (11), (12), we have $T(\tilde{\alpha}_1) = T(\tilde{\alpha}_2)$, $G(\tilde{\alpha}_1) > G(\tilde{\alpha}_2)$, hence $\tilde{\alpha}_1 \succ \tilde{\alpha}_2$. These results remain to be further discussed.

In general, the risk attitude of the decision maker is a very important factor to measure the crispness of IVIFNs. In Example 1, the risk-seeking decision maker believes $\tilde{\alpha}_1 \succ \tilde{\alpha}_2$, the risk-averse decision maker believes $\tilde{\alpha}_1 \prec \tilde{\alpha}_2$ and the risk-neutral believes $\tilde{\alpha}_1 \sim \tilde{\alpha}_2$.

In the decision making process, the decision result depends on risk attitude of the decision maker. Here, we introduce the risk parameter and a new score function with a new accuracy function proposed based on the risk parameter.

Firstly, we introduce $\lambda \in [0, 1]$ to reflect the makers' attitudes in determining the membership degree and the non-membership degree. For an IVIFN $\tilde{\alpha} = ([a, b], [c, d])$, letting

$$M^{\lambda}(\mu_{\tilde{\alpha}}) = a + \lambda(b - a), M^{\lambda}(\nu_{\tilde{\alpha}}) = c + (1 - \lambda)(d - c),$$

$$M^{\lambda}(\pi_{\tilde{\alpha}}) = 1 - M^{\lambda}(\mu_{\tilde{\alpha}}) - M^{\lambda}(\nu_{\tilde{\alpha}}) = \lambda(1 - b - c) + (1 - \lambda)(1 - a - d),$$

we have $M^{\lambda}(\mu_{\tilde{\alpha}}) \in [a, b]$, $M^{\lambda}(\nu_{\tilde{\alpha}}) \in [c, d]$, $M^{\lambda}(\pi_{\tilde{\alpha}}) \in [1-b-d, 1-a-c]$. If $0 \leq \lambda < \frac{1}{2}$, the maker is risk-averse. If $\lambda = \frac{1}{2}$ , he is risk-neutral. If $\frac{1}{2} < \lambda \leq 1$, he is risk-seeking.

Secondly, for the hesitancy interval, according to the vote model and considering the hesitancy people are always affected by supporters and opposers and tend to support and oppose respectively, we define a new score function and accuracy function.

**Definition 3.7** *Let $\tilde{\alpha} = ([a, b], [c, d])$ be an IVIFN. Its score function and accuracy function can be denoted as:*

$$S^{\lambda}(\tilde{\alpha}) = [M^{\lambda}(\mu_{\tilde{\alpha}}) + M^{\lambda}(\mu_{\tilde{\alpha}})M^{\lambda}(\pi_{\tilde{\alpha}})] - [M^{\lambda}(\nu_{\tilde{\alpha}}) + M^{\lambda}(\nu_{\tilde{\alpha}})M^{\lambda}(\pi_{\tilde{\alpha}})], \tag{13}$$

$$h^{\lambda}(\tilde{\alpha}) = [M^{\lambda}(\mu_{\tilde{\alpha}}) + M^{\lambda}(\mu_{\tilde{\alpha}})M^{\lambda}(\pi_{\tilde{\alpha}})] + [M^{\lambda}(\nu_{\tilde{\alpha}}) + M^{\lambda}(\nu_{\tilde{\alpha}})M^{\lambda}(\pi_{\tilde{\alpha}})], \tag{14}$$

*where*

$$M^{\lambda}(\mu_{\tilde{\alpha}}) = a + \lambda(b - a), M^{\lambda}(\nu_{\tilde{\alpha}}) = c + (1 - \lambda)(d - c),$$

$$M^{\lambda}(\pi_{\tilde{\alpha}}) = \lambda(1 - b - c) + (1 - \lambda)(1 - a - d), \lambda \in [0, 1].$$

The score function and accuracy function can be rewritten as

$$S^{\lambda}(\tilde{\alpha}) = [M^{\lambda}(\mu_{\tilde{\alpha}}) - M^{\lambda}(\nu_{\tilde{\alpha}})](1 + M^{\lambda}(\pi_{\tilde{\alpha}})), \tag{15}$$

$$h^{\lambda}(\tilde{\alpha}) = [M^{\lambda}(\mu_{\tilde{\alpha}}) + M^{\lambda}(\nu_{\tilde{\alpha}})](1 + M^{\lambda}(\pi_{\tilde{\alpha}})). \tag{16}$$

For the IVIFNs in Example 1,

$$\tilde{\alpha}_1 = ([0.3, 0.5], [0.2, 0.4]), \tilde{\alpha}_2 = ([0.35, 0.45], [0.25, 0.35]),$$

according to Eq. (15), we obtain

$$S^{\lambda}(\tilde{\alpha}_1) = 1.3(0.4\lambda - 0.1), S^{\lambda}(\tilde{\alpha}_2) = 1.3(0.2\lambda),$$

then $S^{\lambda}(\tilde{\alpha}_1) - S^{\lambda}(\tilde{\alpha}_2) = 0.26(\lambda - \frac{1}{2})$. So, we have

1) If $\lambda < \frac{1}{2}$, then $S^{\lambda}(\tilde{\alpha}_1) < S^{\lambda}(\tilde{\alpha}_2)$, the risk-averse believes $\tilde{\alpha}_1 \prec \tilde{\alpha}_2$.

2) If $\lambda > \frac{1}{2}$, then $S^{\lambda}(\tilde{\alpha}_1) > S^{\lambda}(\tilde{\alpha}_2)$, the risk-seeking regards $\tilde{\alpha}_1 \succ \tilde{\alpha}_2$.

3) If $\lambda = \frac{1}{2}$, then $S^{\lambda}(\tilde{\alpha}_1) = S^{\lambda}(\tilde{\alpha}_2)$, the risk-neutral thinks $\tilde{\alpha}_1 \sim \tilde{\alpha}_2$.

From the above results, we can see that the derived rankings maybe different due to the different risk attitudes of the decision maker. This conclusion satisfies the real situation. By using the methods proposed by Lakshmana [22] and Wang [23], we obtain that $\tilde{\alpha}_1 \succ \tilde{\alpha}_2$ which belongs to the risk-seeker's decision result.

Based on the new score function and accuracy function, a prioritized comparison method is introduced as follows.

**Definition 3.8** *Let $\tilde{\alpha}$ and $\tilde{\beta}$ be two IVIFNs. Then:*

1) *If $S^{\lambda}(\tilde{\alpha}) < S^{\lambda}(\tilde{\beta})$, then $\tilde{\alpha}$ is smaller than $\tilde{\beta}$, denoted by $\tilde{\alpha} \prec \tilde{\beta}$.*

2) *If $S^{\lambda}(\tilde{\alpha}) > S^{\lambda}(\tilde{\beta})$, then $\tilde{\alpha}$ is larger than $\tilde{\beta}$, shown as $\tilde{\alpha} \succ \tilde{\beta}$.*

3) *If $S^{\lambda}(\tilde{\alpha}) = S^{\lambda}(\tilde{\beta})$, then :*

i) *if $h^{\lambda}(\tilde{\alpha}) < h^{\lambda}(\tilde{\beta})$, $\tilde{\alpha}$ is smaller than $\tilde{\beta}$, denoted by $\tilde{\alpha} \prec \tilde{\beta}$;*

ii) *if $h^{\lambda}(\tilde{\alpha}) > h^{\lambda}(\tilde{\beta})$, $\tilde{\alpha}$ is larger than $\tilde{\beta}$, denoted by $\tilde{\alpha} \succ \tilde{\beta}$;*

iii) *if $h^{\lambda}(\tilde{\alpha}) = h^{\lambda}(\tilde{\beta})$, $\tilde{\alpha}$ is equivalent $\tilde{\beta}$, shown as $\tilde{\alpha} \sim \tilde{\beta}$.*

In order to show that the ranking method based on the new score function and accuracy function is more reasonable than the existing methods proposed by Xu [19], Ye [20], Lee [21], Lakshmana [22] and Wang [23], some illustrative examples are given as follows.

*Example 2* Let $\tilde{\alpha}_3 = ([0.2, 0.3], [0.6, 0.7])$ and $\tilde{\alpha}_4 = ([0.3, 0.4], [0.4, 0.6])$ be two IVIFVs for two alternatives. Clearly, $\tilde{\alpha}_3 \prec \tilde{\alpha}_4$.

Using the ranking method proposed by Xu [19], Ye [20], Lee [21], Lakshmana [22] and Wang [23], we can obtain that $\tilde{\alpha}_3 \prec \tilde{\alpha}_4$. By applying the ranking method in this section, we obtain that $S^{\lambda}(\tilde{\alpha}_3) < S^{\lambda}(\tilde{\alpha}_4)$ for any $\lambda, \lambda \in [0, 1]$. So, the makers with different risk attitude all believe $\tilde{\alpha}_3 \prec \tilde{\alpha}_4$, which is reasonable.

**Remark 1** Example 2 demonstrates the feasibility of the proposed ranking method for two general IVIFVs.

*Example 3* Let $\tilde{\alpha}_5 = ([0, 0.2], [0.6, 0.8])$ and $\tilde{\alpha}_6 = ([0.3, 0.4], [0.1, 0.1])$ be two IVIFVs for two alternatives. Clearly, $\tilde{\alpha}_5 \prec \tilde{\alpha}_6$.

Using the ranking method proposed by Xu [19], Lee [21] and Lakshmana [22], we can obtain that $\tilde{\alpha}_5 \prec \tilde{\alpha}_6$. By applying the ranking method in this section, we obtain that $S^\lambda(\tilde{\alpha}_5) < S^\lambda(\tilde{\alpha}_6)$ for any $\lambda, \lambda \in [0, 1]$. So, the decision makers all believe $\tilde{\alpha}_5 \prec \tilde{\alpha}_6$, which is reasonable. By applying Eqs. (11), (12), we have $T(\tilde{\alpha}_5) < T(\tilde{\alpha}_6)$, $G(\tilde{\alpha}_5) > G(\tilde{\alpha}_6)$. Using the ranking method proposed by Ye [20], we can obtain that $\tilde{\alpha}_5 \succ \tilde{\alpha}_6$, which is contrary to the fact.

**Remark 2** Example 3 shows that the ranking method presented by Ye [20] cannot give the correct order of the two alternatives and the ranking method presented in this section can overcome the invalidity of Ye's ranking method.

*Example 4* Let $\tilde{\alpha}_7 = ([0.45, 0.55], [0.25, 0.35])$ and $\tilde{\alpha}_8 = ([0.4, 0.6], [0.2, 0.4])$ be two IVIFVs for two alternatives.

Using the ranking method proposed by Xu [19], Ye [20] and Lee [21], we can obtain that $\tilde{\alpha}_7 \sim \tilde{\alpha}_8$, so we cannot get the order of these two alternatives. By applying the ranking method proposed by Lakshmana [22], we can obtain that $\tilde{\alpha}_7 \prec \tilde{\alpha}_8$. By applying Eqs. (11), (12), we have $T(\tilde{\alpha}_7) = T(\tilde{\alpha}_8)$, $G(\tilde{\alpha}_7) < G(\tilde{\alpha}_8)$, so $\tilde{\alpha}_7 \prec \tilde{\alpha}_8$. Using the ranking method in this section, we obtain that

$$S^\lambda(\tilde{\alpha}_7) = 1.2(0.1 + 0.2\lambda), \quad S^\lambda(\tilde{\alpha}_8) = 1.2(0.4\lambda),$$

then $S^\lambda(\tilde{\alpha}_7) - S^\lambda(\tilde{\alpha}_8) = 0.24(\frac{1}{2} - \lambda)$. So, we have

1) If $\lambda < \frac{1}{2}$, then $S^\lambda(\tilde{\alpha}_7) > S^\lambda(\tilde{\alpha}_8)$, the risk-averse thinks $\tilde{\alpha}_7 \succ \tilde{\alpha}_8$.

2) If $\lambda > \frac{1}{2}$, then $S^\lambda(\tilde{\alpha}_7) < S^\lambda(\tilde{\alpha}_8)$, the risk-seeking believes $\tilde{\alpha}_7 \prec \tilde{\alpha}_8$.

3) If $\lambda = \frac{1}{2}$, then $S^\lambda(\tilde{\alpha}_7) = S^\lambda(\tilde{\alpha}_8)$, the risk-neutral regards $\tilde{\alpha}_7 \sim \tilde{\alpha}_8$.

**Remark 3** Example 4 illustrates that the derived rankings may be different due to the different risk attitudes of the decision maker. This conclusion satisfies the real situation. By using Eqs. (6)-(9), we obtain that $\tilde{\alpha}_7 \sim \tilde{\alpha}_8$, so we cannot get the order of these two alternatives. Using the ranking method proposed by Lakshmana [22] and Wang [23], we obtain $\tilde{\alpha}_7 \prec \tilde{\alpha}_8$, which belongs to the risk-seeker's decision result. Additionally, we can give some properties of the score function $S^\lambda(\tilde{\alpha})$.

**Theorem 3.1** *Let $\tilde{\alpha} = ([a, b], [c, d])$ be an IVIFN. If the score function $S^\lambda(\tilde{\alpha})$ is defined by Definition* 3.7*, then*

1) $-1 \leq S^\lambda(\tilde{\alpha}) \leq 1$ *for any* $\lambda \in [0, 1]$.

2) *For any* $\lambda \in [0, 1]$, $S^\lambda(\tilde{\alpha}) = 1$ *if and only if* $\tilde{\alpha} = ([1, 1], [0, 0])$.

3) *For any* $\lambda \in [0, 1]$, $S^\lambda(\tilde{\alpha}) = -1$ *if and only if* $\tilde{\alpha} = ([0, 0], [1, 1])$.

*Proof* Combining Eq. (15) with $M^{\lambda}(\pi_{\tilde{\alpha}}) = 1 - M^{\lambda}(\mu_{\tilde{\alpha}}) - M^{\lambda}(\nu_{\tilde{\alpha}})$, we see that

$$S^{\lambda}(\tilde{\alpha}) = [1 - M^{\lambda}(\nu_{\tilde{\alpha}})]^2 - [1 - M^{\lambda}(\mu_{\tilde{\alpha}})]^2. \tag{17}$$

It is noted that

$$0 \le 1 - M^{\lambda}(\mu_{\tilde{\alpha}}) \le 1, \;\; 0 \le 1 - M^{\lambda}(\nu_{\tilde{\alpha}}) \le 1,$$

hence

$$-1 \le S^{\lambda}(\tilde{\alpha}) \le 1, \forall \lambda \in [0, 1].$$

If $S^{\lambda}(\tilde{\alpha}) = 1$, according to Eq. (17), we have

$$M^{\lambda}(\nu_{\tilde{\alpha}}) = c + (1 - \lambda)(d - c) = 0, \;\; M^{\lambda}(\mu_{\tilde{\alpha}}) = a + \lambda(b - a) = 1.$$

Since $\lambda$ is a random number, then

$$d = c = 0, \;\; b = a = 1.$$

On the other hand, if $\tilde{\alpha} = ([1, 1], [0, 0])$, then it is easy to prove that $S^{\lambda}(\tilde{\alpha}) = 1$ for any $\lambda \in [0, 1]$. Hence

$$S^{\lambda}(\tilde{\alpha}) = 1 \Leftrightarrow \tilde{\alpha} = ([1, 1], [0, 0]), \forall\, \lambda \in [0, 1].$$

Similar to the previous proof method, if $S^{\lambda}(\tilde{\alpha}) = -1$, according to Eq. (17), we have

$$M^{\lambda}(\nu_{\tilde{\alpha}}) = c + (1 - \lambda)(d - c) = 1, \;\; M^{\lambda}(\mu_{\tilde{\alpha}}) = a + \lambda(b - a) = 0.$$

Since $\lambda$ is a random number, then

$$d = c = 1, \;\; b = a = 0.$$

On the other hand, if $\tilde{\alpha} = ([0, 0], [1, 1])$ , then it is easy to prove that $S^{\lambda}(\tilde{\alpha}) = -1$ for any $\lambda \in [0, 1]$. Hence

$$S^{\lambda}(\tilde{\alpha}) = -1 \Leftrightarrow \tilde{\alpha} = ([0, 0], [1, 1]), \;\forall \in [0, 1].$$

**Theorem 3.2** *Let* $\tilde{\alpha} = ([a_1, b_1], [c_1, d_1])$ *and* $\tilde{\beta} = ([a_2, b_2], [c_2, d_2])$ *be two IVIFNs. If* $a_1 > a_2, b_1 > b_2$ *and* $c_1 < c_2, d_1 < d_2$*, then* $S^{\lambda}(\tilde{\alpha}) > S^{\lambda}(\tilde{\beta})$*,* $\forall\, \lambda \in [0, 1]$.

*Proof* Since $a_1 > a_2, b_1 > b_2$ and $c_1 < c_2, d_1 < d_2$ , then

$$1 - M^{\lambda}(\mu_{\tilde{\alpha}}) < 1 - M^{\lambda}(\mu_{\tilde{\beta}}), 1 - M^{\lambda}(\nu_{\tilde{\alpha}}) > 1 - M^{\lambda}(\nu_{\tilde{\beta}}),$$

according to Eq. (17), we can get

$$S^{\lambda}(\tilde{\alpha}) > S^{\lambda}(\tilde{\beta}),\ \forall\lambda \in [0,1].$$

**Theorem 3.3** *Let $\tilde{\alpha} = ([a_1, b_1], [c_1, d_1])$ and $\tilde{\beta} = ([a_2, b_2], [c_2, d_2])$ be two IVIFNs. If $S^{\lambda}(\tilde{\alpha}) = S^{\lambda}(\tilde{\beta}), h^{\lambda}(\tilde{\alpha}) = h^{\lambda}(\tilde{\beta})$, then $\tilde{\alpha} = \tilde{\beta}$ for any $\lambda \in [0,1]$.*

*Proof* Since $S^{\lambda}(\tilde{\alpha}) = S^{\lambda}(\tilde{\beta}), h^{\lambda}(\tilde{\alpha}) = h^{\lambda}(\tilde{\beta})$, then according to Eqs. (15) and (16), we can get

$$[M^{\lambda}(\mu_{\tilde{\alpha}}) - M^{\lambda}(\nu_{\tilde{\alpha}})](1 + M^{\lambda}(\pi_{\tilde{\alpha}})) = [M^{\lambda}(\mu_{\tilde{\beta}}) - M^{\lambda}(\nu_{\tilde{\beta}})](1 + M^{\lambda}(\pi_{\tilde{\beta}})), \tag{18}$$

$$[M^{\lambda}(\mu_{\tilde{\alpha}}) + M^{\lambda}(\nu_{\tilde{\alpha}})](1 + M^{\lambda}(\pi_{\tilde{\alpha}})) = [M^{\lambda}(\mu_{\tilde{\beta}}) + M^{\lambda}(\nu_{\tilde{\beta}})](1 + M^{\lambda}(\pi_{\tilde{\beta}})). \tag{19}$$

Hence

$$M^{\lambda}(\mu_{\tilde{\alpha}})(1 + M^{\lambda}(\pi_{\tilde{\alpha}})) = M^{\lambda}(\mu_{\tilde{\beta}})(1 + M^{\lambda}(\pi_{\tilde{\beta}})), \tag{20}$$

$$M^{\lambda}(\nu_{\tilde{\alpha}})(1 + M^{\lambda}(\pi_{\tilde{\alpha}})) = M^{\lambda}(\nu_{\tilde{\beta}})(1 + M^{\lambda}(\pi_{\tilde{\beta}})). \tag{21}$$

Then it is obvious that

$$M^{\lambda}(\mu_{\tilde{\alpha}})M^{\lambda}(\nu_{\tilde{\beta}}) = M^{\lambda}(\mu_{\tilde{\beta}})M^{\lambda}(\nu_{\tilde{\alpha}}). \tag{22}$$

Eq. (22) also can be rewritten as

$$\begin{aligned} &a_1d_2 + \lambda(-2a_1d_2 + a_1c_2 + b_1d_2) + \lambda^2(a_1d_2 - a_1c_2 - b_1d_2 + b_1c_2) \\ &\quad = a_2d_1 + \lambda(-2a_2d_1 + a_2c_1 + b_2d_1) + \lambda^2(a_2d_1 - a_2c_1 - b_2d_1 + b_2c_1). \end{aligned}$$

Since $\lambda$ is a random number in $[0,1]$, then

$$a_1d_2 = a_2d_1,$$

$$-2a_1d_2 + a_1c_2 + b_1d_2 = -2a_2d_1 + a_2c_1 + b_2d_1,$$

$$a_1d_2 - a_1c_2 - b_1d_2 + b_1c_2 = a_2d_1 - a_2c_1 - b_2d_1 + b_2c_1.$$

It is easy to see that

$$a_1d_2 = a_2d_1, \tag{23}$$

$$b_1c_2 = b_2c_1, \tag{24}$$

$$a_1c_2 + b_1d_2 = a_2c_1 + b_2d_1. \tag{25}$$

Similar to the previous proof method, according to Eqs. (18) and (19), we also have

$$2a_1 - 2d_1 - a_1^2 + d_1^2 = 2a_2 - 2d_2 - a_2^2 + d_2^2, \tag{26}$$

$$\begin{aligned} &-a_1 + b_1 - c_1 + d_1 + a_1^2 - d_1^2 - a_1b_1 + c_1d_1 \\ &\quad = -a_2 + b_2 - c_2 + d_2 + a_2^2 - d_2^2 - a_2b_2 + c_2d_2, \end{aligned} \tag{27}$$

$$2a_1 + 2d_1 - a_1^2 - d_1^2 - 2a_1d_1 = 2a_2 + 2d_2 - a_2^2 - d_2^2 - 2a_2d_2, \tag{28}$$

$$\begin{aligned} &a_1^2 + d_1^2 - a_1b_1 - c_1d_1 - a_1c_1 + 2a_1d_1 - b_1d_1 \\ &\quad = a_2^2 + d_2^2 - a_2b_2 - c_2d_2 - a_2c_2 + 2a_2d_2 - b_2d_2. \end{aligned} \tag{29}$$

Next, based on above analysis, we will split the argument into two cases.

*Case I* $a_1 = b_1, c_1 = d_1$ and $a_2 = b_2, c_2 = d_2$ ($\tilde{\alpha}, \tilde{\beta}$ are two IFNs).

According to Eqs. (26) and (28), we have

$$(a_1 - c_1)(a_1 + c_1 - 2) = (a_2 - c_2)(a_2 + c_2 - 2),$$

$$a_1 + c_1 - 1 = \pm(a_2 + c_2 - 1).$$

Since $a_i + c_i \in [0, 1]$ $(i = 1, 2)$, then $a_1 = a_2, c_1 = c_2$. Hence

$$\tilde{\alpha} = \tilde{\beta}.$$

*Case II* $\tilde{\alpha}$ and $\tilde{\beta}$ is at least one IVIFN, assume that $\tilde{\beta}$ is an IVIFN, then $a_2 \neq b_2$ or $c_2 \neq d_2$. We proceed stepwise as follows:

i) When $d_2 = 0$;

Since $0 \leq c_2 \leq d_2, 0 \leq a_2 \leq b_2$, then $c_2 = d_2 = 0, a_2 \neq b_2, b_2 \neq 0$.

According to Eqs. (24) and (25), we have

$$c_1 = 0, \quad d_1 = 0.$$

Hence

$$c_1 = c_2 = 0, \quad d_1 = d_2 = 0.$$

So, according to Eq. (26), we have

$$a_1 - 1 = \pm(a_2 - 1).$$

Since $a_i \in [0, 1]$ $(i = 1, 2)$, then $a_1 = a_2$.

From Eq. (27), we have

$$b_1(1 - a_1) = b_2(1 - a_2).$$

Since $a_i \leq b_i \leq 1$ $(i = 1, 2)$, then it is easy to prove that $b_1 = b_2$.
Hence

$$\tilde{\alpha} = \tilde{\beta}.$$

ii) At $b_2 = 0$, the proof is similar to that of i), we have

$$\tilde{\alpha} = \tilde{\beta}.$$

iii) When $d_2 \neq 0$ and $b_2 \neq 0$, according to Eqs. (23) and (24), we have

$$a_1 = \frac{d_1}{d_2} a_2,\ c_1 = \frac{b_1}{b_2} c_2.$$

According to Eq. (25), we have

$$(\frac{d_1}{d_2} - \frac{b_1}{b_2}) a_2 c_2 = (\frac{d_1}{d_2} - \frac{b_1}{b_2}) b_2 d_2.$$

Assume that $\frac{d_1}{d_2} \neq \frac{b_1}{b_2}$, then $a_2 c_2 = b_2 d_2, a_2 \neq 0, c_2 \neq 0$. Since $0 < a_2 \leq b_2, 0 < c_2 \leq d_2$, $a_2 \neq b_2$ (or $c_2 \neq d_2$), then $a_2 c_2 < b_2 d_2$, this is contrary to the conclusion $a_2 c_2 = b_2 d_2$. Hence $\frac{d_1}{d_2} = \frac{b_1}{b_2}$.

Thus

$$a_1 = \frac{b_1}{b_2} a_2,\ c_1 = \frac{b_1}{b_2} c_2,\ b_1 = \frac{b_1}{b_2} b_2,\ d_1 = \frac{b_1}{b_2} d_2. \tag{30}$$

So

$$M^\lambda(\mu_{\tilde{\alpha}}) = a_1 + \lambda(b_1 - a_1) = \frac{b_1}{b_2} a_2 + \lambda(\frac{b_1}{b_2} b_2 - \frac{b_1}{b_2} a_2) = \frac{b_1}{b_2} M^\lambda(\mu_{\tilde{\beta}}),$$

$$M^\lambda(\nu_{\tilde{\alpha}}) = c_1 + (1-\lambda)(d_1 - c_1) = \frac{b_1}{b_2} c_2 + (1-\lambda)(\frac{b_1}{b_2} d_2 - \frac{b_1}{b_2} c_2) = \frac{b_1}{b_2} M^\lambda(\nu_{\tilde{\beta}}).$$

According to Eq. (20), we have

$$2(\frac{b_1}{b_2} - 1) = (\frac{b_1}{b_2} - 1)(\frac{b_1}{b_2} + 1)(M^\lambda(\mu_{\tilde{\beta}}) + M^\lambda(\nu_{\tilde{\beta}})).$$

Assume that $\frac{b_1}{b_2} \neq 1$, then

$$M^\lambda(\mu_{\tilde{\beta}}) + M^\lambda(\nu_{\tilde{\beta}}) = \frac{2}{\frac{b_1}{b_2} + 1},$$

$$M^{\lambda}(\mu_{\tilde{\alpha}}) + M^{\lambda}(\nu_{\tilde{\alpha}}) = \frac{b_1}{b_2}(M^{\lambda}(\mu_{\tilde{\beta}}) + M^{\lambda}(\nu_{\tilde{\beta}})) = \frac{\frac{2b_1}{b_2}}{\frac{b_1}{b_2}+1}.$$

Since $M^{\lambda}(\mu_{\tilde{\alpha}}) + M^{\lambda}(\nu_{\tilde{\alpha}}) \leq 1$, $M^{\lambda}(\mu_{\tilde{\beta}}) + M^{\lambda}(\nu_{\tilde{\beta}}) \leq 1$, then $\frac{b_1}{b_2} = 1$, this is contrary to assumption $\frac{b_1}{b_2} \neq 1$. Hence $\frac{b_1}{b_2} = 1$.

So, according to Eq. (30), we have

$$a_1 = a_2,\ b_1 = b_2,\ c_1 = c_2,\ d_1 = d_2.$$

Thus

$$\tilde{\alpha} = \tilde{\beta}.$$

The proof is completed.

## 4. Multi-criteria Decision Making Based on Interval-valued Intuitionistic Fuzzy Information

In this section, we present a handing method to a multi-criteria decision making problem with weights.

Let $A = \{A_1, A_2, \cdots, A_n\}$ be a set of alternatives and $G = \{G_1, G_2, \cdots, G_m\}$ be a set of criteria. Assume that the weight of criterion $G_j$ ($j = 1, 2, \cdots, m$), entered by the decision-maker, is $w_j, w_j \in [0, 1]$ and $\sum\limits_{j=1}^{m} w_j = 1$. In this case, the evaluation of the alternatives $A_i$ with respect to the criterion $G_j$ is an IVIFN represented by$\tilde{r}_{ij} = ([a'_{ij}, b'_{ij}], [c'_{ij}, d'_{ij}])(i = 1, 2, \cdots, n; j = 1, 2, \cdots, m)$, which indicates the degree that the alternative $A_i$ satisfies or does not satisfy the criterion $G_j$. $\tilde{r}_{ij}$ given by decision makers or experts. Therefore, we can elicit a decision matrix $D = (\tilde{r}_{ij})_{n\times m}$, which is expressed by IVIFNs.

In summary, the multi-criteria decision making procedure designed to find the best alternative is given by the following steps:

**Step 1**: Normalized $D = (\tilde{r}_{ij})_{n\times m}$ into the IVIFN decision matrix $\bar{D} = (\tilde{\alpha}_{ij})_{n\times m}$, where

$$\begin{aligned}\tilde{\alpha}_{ij} &= ([a_{ij}, b_{ij}], [c_{ij}, d_{ij}])\\ &= \begin{cases} ([a'_{ij}, b'_{ij}], [c'_{ij}, d'_{ij}]), \text{for benefit criterion } G_j, \\ ([c'_{ij}, d'_{ij}], [a'_{ij}, b'_{ij}]), \text{for cost criterion } G_j, \end{cases} \quad i = 1, 2, \cdots, n.\end{aligned}$$

The normalization formula of the IVIFN decision matrix was introduced by Xu and Hu [25]. Let $w = (w_2, w_2, \cdots, w_m)^T$ be the relative weights vector of all criteria.

**Step 2**: According to the decision maker's risk attitude, choose the attitude parameter $\lambda, \lambda \in [0, 1]$. Then calculate the weighted comprehensive score value $S_i$ and the weighted comprehensive accuracy value $h_i$ of alternatives $A_i$ ($i = 1, 2, \cdots, n$) by the following formulas:

$$S_i(\tilde{\alpha}_{i1}, \tilde{\alpha}_{i2}, \cdots, \tilde{\alpha}_{im}) = w_1 S^{\lambda}(\tilde{\alpha}_{i1}) + w_2 S^{\lambda}(\tilde{\alpha}_{i2}) + \cdots + w_m S^{\lambda}(\tilde{\alpha}_{im}), \tag{31}$$

$$h_i(\tilde{\alpha}_{i1}, \tilde{\alpha}_{i2}, \cdots, \tilde{\alpha}_{im}) = w_1 h^{\lambda}(\tilde{\alpha}_{i1}) + w_2 h^{\lambda}(\tilde{\alpha}_{i2}) + \cdots + w_m h^{\lambda}(\tilde{\alpha}_{im}). \tag{32}$$

**Step 3**: Rank the alternatives $A_i$ $(i = 1, 2, \cdots, n)$ and select the best one(s) in accordance with the value $S_i, h_i$ $(i = 1, 2, \cdots, n)$. The larger the value of $S_i$ is, the better the alternative is; in case that they are equal, we further compare their accuracy value $h_i$ to form their ranks, the larger the value of $h_i$ is, the better the alternative is.

## 5. Illustrative Example

In this section, we show the application of the proposed fuzzy decision-making method through a practical example [26].

There is a panel with four possible alternatives to invest: $A_1$ is a car company; $A_2$ serves as a food company; $A_3$ denotes a computer company; $A_4$ refers to an arms company. The investment company must make a decision according to the following five criteria: 1) $G_1$ stands for the productivity; 2) $G_2$ represents the technological innovation capability; 3) $G_3$ serves as the marketing capability; 4) $G_4$ refers to the management; 5) $G_5$ denotes the risk avoidance. The criteria are independent and the criteria weights comprise a vector $w = (0.2, 0.3, 0.15, 0.1, 0.25)$.

**Step 1**: The normalized IVIFN decision matrix $\bar{D} = (\tilde{\alpha}_{ij})_{4\times5}$ can be listed as follows:

$$\bar{D} = \begin{pmatrix} ([0.4,0.5],[0.1,0.3]) & ([0.5,0.6],[0.1,0.2]) & ([0.3,0.4],[0.2,0.3]) & ([0.7,0.8],[0.1,0.2]) & ([0.5,0.6],[0.1,0.2]) \\ ([0.5,0.6],[0.1,0.2]) & ([0.3,0.4],[0.1,0.3]) & ([0.7,0.8],[0.1,0.2]) & ([0.3,0.4],[0.3,0.4]) & ([0.4,0.5],[0.1,0.2]) \\ ([0.6,0.7],[0.1,0.2]) & ([0.7,0.8],[0.1,0.2]) & ([0.5,0.6],[0.3,0.4]) & ([0.4,0.5],[0.3,0.4]) & ([0.3,0.5],[0.3,0.4]) \\ ([0.5,0.6],[0.2,0.3]) & ([0.4,0.5],[0.3,0.4]) & ([0.6,0.7],[0.2,0.3]) & ([0.6,0.7],[0.2,0.3]) & ([0.6,0.7],[0.1,0.3]) \end{pmatrix}.$$

**Step 2**: According to his risk attitude, we choose the attitude parameter $\lambda, \lambda \in [0, 1]$. Then, we calculate the weighted comprehensive score values of alternatives $A_i$ $(i = 1, 2, 3, 4)$, Table 1 shows the results obtained by Eq. (31).

Table 1: The score values of different attitude parameters.

| Attitude parameter | Score value |
|---|---|
| $\lambda = \frac{1}{3}$ | $S_1 = 0.3918,\ S_2 = 0.3208,\ S_3 = 0.3345,\ S_4 = 0.3022$ |
| $\lambda = \frac{1}{2}$ | $S_1 = 0.4405,\ S_2 = 0.3733,\ S_3 = 0.3786,\ S_4 = 0.3474$ |
| $\lambda = \frac{2}{3}$ | $S_1 = 0.4895,\ S_2 = 0.4262,\ S_3 = 0.4223,\ S_4 = 0.3930$ |

**Step 3**: Rank the alternatives $A_i$ $(i = 1, 2, 3, 4)$, and the orders of different attitude parameters are shown in Table 2.

For the convenience of comparing, the ranking orders of different methods are shown in Table 2.

Table 2: Ranking orders of the alternatives for different methods.

| Method | | Ranking order |
|---|---|---|
| Xu's: $S(\tilde{\alpha}) = (a - c + b - d)/2, h(\tilde{\alpha}) = (a + b + c + d)/2$ | | $A_1 \succ A_3 \succ A_4 \succ A_2$ |
| Ye's: $S_Y(\tilde{\alpha}) = a + b - 1 + (c + d)/2$ | | $A_3 \succ A_4 \succ A_1 \succ A_2$ |
| Lee's: $S_L(\tilde{\alpha}) = (2 + a + b - c - d)/(3 - a - b - c - d)$ | | $A_3 \succ A_4 \succ A_1 \succ A_2$ |
| Lakshmana's: $S_A(\tilde{\alpha}) = [a + b - d(1 - b) - c(1 - a)]/2$ | | $A_3 \succ A_4 \succ A_1 \succ A_2$ |
| Wang's: $S(\tilde{\alpha}) = (a - c + b - d)/2, h(\tilde{\alpha}) = (a + b + c + d)/2,$ $T(\tilde{\alpha}) = b + c - a - d, G(\tilde{\alpha}) = b + d - a - c.$ | | $A_1 \succ A_3 \succ A_4 \succ A_2$ |
| Proposed method: $S^{\lambda}(\tilde{\alpha})$ | $\lambda = \frac{1}{3}$ | $A_1 \succ A_3 \succ A_2 \succ A_4$ |
| | $\lambda = \frac{2}{3}$ | $A_1 \succ A_2 \succ A_3 \succ A_4$ |
| | $\lambda = \frac{1}{2}$ | $A_1 \succ A_3 \succ A_2 \succ A_4$ |

Therefore, from Table 2 we can see that the ranking order may be different due to different risk attitudes of a decision maker. And the risk-averse thinks $A_1 \succ A_3 \succ A_2 \succ A_4$, the risk-seeking believes $A_1 \succ A_2 \succ A_3 \succ A_4$, the risk-neutral regards $A_1 \succ A_3 \succ A_2 \succ A_4$. The decision makers with different risk attitude all believe $A_1 \succ A_2$, which satisfies the real situation. But the ranking order obtained by the proposed method is remarkably different from those obtained by Xu [19], Ye [20], Lee [21], Lakshmana[22] and Wang [23]. The ranking orders may differ according to various methods because different algorithms have variety points of view. The advantage of the proposed method over the other five methods is that it can provide the decision maker with more selecting schemes according to their risk attitude.

## 6. Conclusion

In this paper, a new score function and accuracy function are proposed to properly reflect a decision maker with risk attitude, based on the discussion of the hesitation of IVIFNs. At the same time, the properties of the new score function are discussed. Besides, a new ranking method of IVIFNs is given. Numerical examples and comparison with other methods illustrate that the method is more reasonable. Furthermore, combined with the interval-valued intuitionistic fuzzy weighted average operator, a multi-criteria decision method based on interval-valued intuitionistic fuzzy environment is developed. Finally, promising numerical results are reported.

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